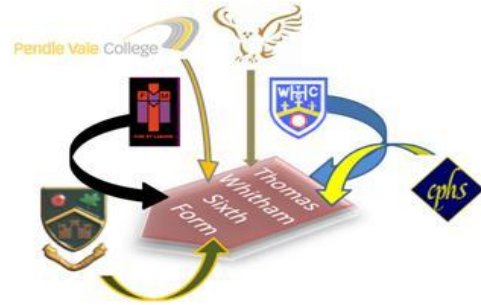




THOMAS WHITHAM SIXTH FORM



Further Mathematics

“eff of ex”

S J Cooper

Wednesday, June 25, 2014

2.2 Definition of a function

2.3 Domain and range of a function

2.10 Sketching of functions

Sketch graphs of linear and quadratic functions

Notation $f(x)$ will be used, eg $f(x) = x^2 - 9$

Domain may be expressed as, for example,

$x > 2$, or 'for all x , except $x = 0$ ' and range may be expressed as $f(x) > -1$

Graphs could be linear, quadratic or restricted to no

more than 3 domains

eg $y = x^2 - 5x + 6$

Label clearly any points of the intersection with the axes

eg A function $f(x)$ is defined as

$$f(x) = x^2 \quad 0 \leq x < 1$$

$$= 1 \quad 1 \leq x < 2$$

$$= 3 - x \quad 2 \leq x < 3$$

Draw the graph of $f(x)$ on the grid below for values of x from 0 to 3

Functions

$f(x)$ which reads “eff of ex” is used to indicate a function of x

A function is simply an expression in terms of x .

Examples

$$f(x) = 2x^2 - x - 3 \quad \text{A quadratic function}$$

$$f(x) = 4x - 5 \quad \text{A linear function}$$

$$f(x) = \sin x \quad \text{A trig function}$$

Example

Given $f(x) = x^2 + 2x - 5$ obtain (i) an expression for $f(a)$

(ii) the value of $f(4)$

(iii) the solutions of the equation $f(a) = 3$.

$$(i) \quad f(a) = \underline{a^2 + 2a - 5}$$

$$(ii) \quad f(4) = 4^2 + 2(4) - 5 \\ = 16 + 8 - 5 \\ = \underline{19}$$

$$(iii) \quad f(a) = 3 \\ a^2 + 2a - 5 = 3 \\ a^2 + 2a - 8 = 0 \\ (a + 4)(a - 2) = 0$$

$$\underline{a = -4} \quad \underline{a = 2}$$

Example

Given $f(x) = 4 - 2x$

(i) Work out $f(3)$

(ii) Solve the equation $f(m) = 7$

$$\begin{aligned} \text{(i)} \quad f(3) &= 4 - 2(3) \\ &= 4 - 6 \\ &= \underline{-2} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad f(m) &= 7 \\ 4 - 2m &= 7 \\ 4 - 7 &= 2m \\ -\frac{3}{2} &= m \end{aligned}$$

Example

$g(x) = ax^2 - 3$ for all values of x .

(a) Given that $g(2) = 13$ find the value of a

(b) Work out the value of $g(4)$

$$\begin{aligned} \text{(a)} \quad g(2) &= 13 \\ a(2)^2 - 3 &= 13 \\ 4a - 3 &= 13 \\ 4a &= 16 \\ \underline{a} &= \underline{4} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad g(4) &= 4(4)^2 - 3 \\ &= 4 \times 16 - 3 \\ &= \underline{61} \end{aligned}$$

Example

$$f(x) = 6x - 8 - x^2 \text{ for all values of } x$$

$$g(x) = (x + 1)(x + a) \text{ for all values of } x$$

(a) Given that $g(3) = 4$, find the value of a

(b) Solve the equation $f(x) = g(x)$

$$\begin{aligned} \text{(a)} \quad g(3) &= 4 \\ (3 + 1)(3 + a) &= 4 \\ 4(3 + a) &= 4 \\ (3 + a) &= 1 \\ \underline{a} &= \underline{-2} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad f(x) &= g(x) \\ 6x - 8 - x^2 &= (x + 1)(x - 2) \\ 6x - 8 - x^2 &= x^2 - x - 2 \\ 0 &= 2x^2 - 7x + 6 \\ 0 &= (2x - 3)(x - 2) \\ \underline{x = \frac{3}{2}, \quad x = 2} \end{aligned}$$

Example

Given $f(x) = x^2 - 3x - 4$ obtain an expression for $f(x + 2)$

$$f(x) = x^2 - 3x - 4$$

$$f(x + 2) = (x + 2)^2 - 3(x + 2) - 4$$

$$f(x + 2) = x^2 + 4x + 4 - 3x - 6 - 4$$

$$\underline{f(x + 2) = x^2 + x - 6}$$

3. Given $f(x) = 4x - 5$.

Obtain an expression for $f(x + 1)$

$$f(x + 1) = \underline{\hspace{10em}}$$

4. Given $g(x) = x^2 - 2x + 1$.

Obtain an expression for $g(x - 2)$

$$g(x - 2) = \underline{\hspace{10em}}$$

5. Given that $f(x) = 4x^2 + 3x - 2$, solve the equation

(a) $f(x) = 5$

$x =$ _____

(b) $f(a) = 3a$

$a =$ _____

6. Given $g(x) = 2x^2 - x + 1$ solve the equation

$$g(a) = g(2a)$$

$a =$ _____

7. Given $f(x) = x^2 + 2x - 3$ solve the equation

$$f(x + 1) = f(x - 1)$$

$x =$ _____

8. The functions f and g are defined as

$$f(x) = 3x^2 - 2$$

$$g(x) = cx - 7$$

Given that $f(3) = g(3)$ find the value of c

$c =$ _____