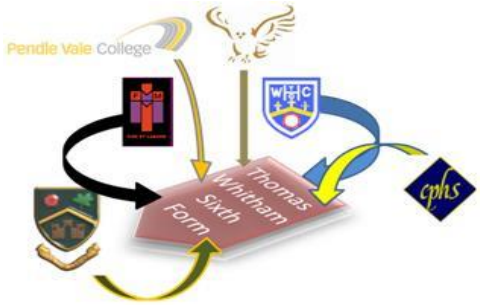
THOMAS WHITHAM SIXTH FORM



Further Mathematics

Calculus Introduction

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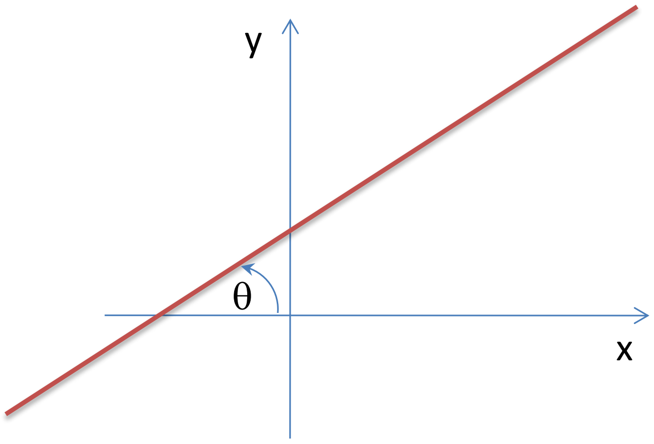
4.1 . Know that the gradient function $\frac{dy}{dx}$ gives the gradient of the curve and measures the rate of change of y with respect to x.

4.2 . Know that the gradient of a function is the gradient of the tangent at that point.

4.3 . Differentiation of kx^n where n is a positive integer or 0, and the sum of such functions.

Gradient of a line

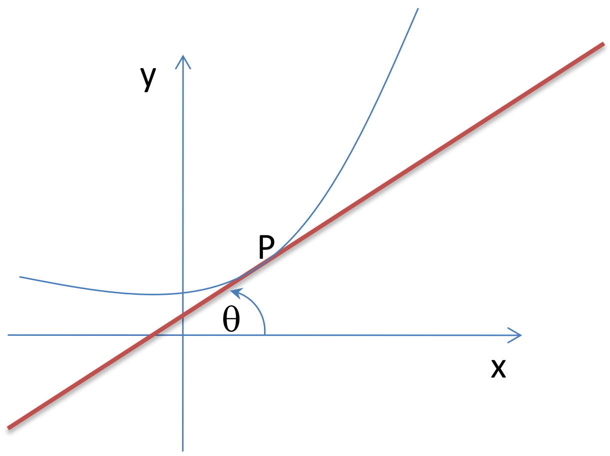


Gradient is given by

$$m = \tan\theta$$

$$\tan\theta = \frac{opp}{Adj}$$

Gradient of a curve at the point P



At P, the gradient is given by the tangent drawn at P.

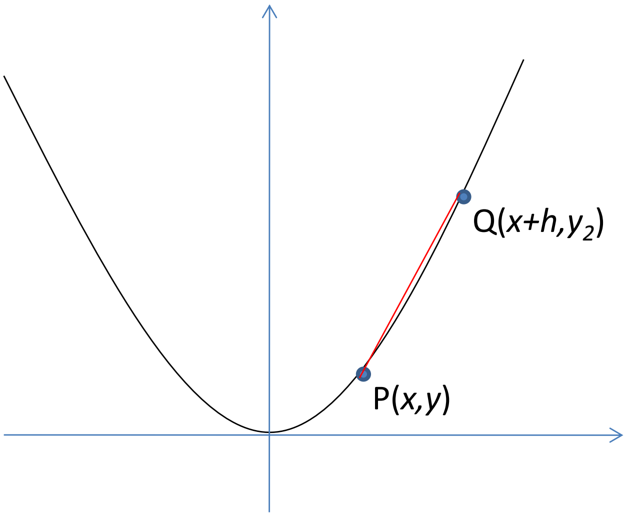
i.e. At P Gradient is given by

$$m = \tan\theta$$

Example

The curve drawn is of $y = x^2$.

Let P be the point with coordinate (x, y).



Gradient of PQ = $\dfrac{y_2 - y}{(x+h) - x}$

$$= \frac{(x+h)^2 - x^2}{h}$$

$$= \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \frac{2xh + h^2}{h}$$

$$= 2x + h$$

However this is the gradient of the chord PQ.

The tangent at P is when P=Q.

That is h = 0

Gradient $= 2x$

We write such a gradient as $\dfrac{dy}{dx}$ hence $\boxed{\dfrac{dy}{dx} = 2x}$

Example

For the curve of $y = x^2 + 3x$ obtain an expression for the gradient of the tangent at the point (x, y)

Gradient of PQ = $\dfrac{y_2 - y}{(x+h) - x}$

$$= \frac{\left[(x+h)^2 + 3(x+h)\right] - \left[x^2 + 3x\right]}{h}$$

$$= \frac{\left[x^2 + 2xh + h^2 + 3x + 3h\right] - \left[x^2 + 3x\right]}{h}$$

$$= \frac{2xh + h^2 + 3h}{h}$$

$$= 2x + h + 3$$

h = 0 $\boxed{\dfrac{dy}{dx} = 2x + 3}$

Example

Obtain an expression for the gradient to the curve at $y = 4x - 1$.

Gradient of PQ = $\frac{y_2 - y}{(x+h) - x}$

$$= \frac{[4(x+h) - 1] - [4x - 1]}{h}$$

$$= \frac{[4x + 4h - 1] - [4x - 1]}{h}$$

$$= \frac{4h}{h}$$

$$= 4$$

$h = 0$ $\quad \frac{dy}{dx} = 4$

Example

Obtain the expansion of $(x + h)^3$

Obtain the gradient to the curve $y = x^3$ at the point (x, y)

$$(x+h)^3 \quad = x^3 + 3x^2h + 3xh^2 + h^3$$

Gradient of PQ = $\frac{y_2 - y}{(x+h) - x}$

$$= \frac{(x+h)^3 - x^3}{h}$$

$$= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^4}{h}$$

$$= \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= 3x^2 + 3xh \ + h^2$$

$h = 0$ $\quad \frac{dy}{dx} = 3x^2$

Example

Obtain the expansion of $(x+h)^4$

Obtain the gradient to the curve $y = x^4$ at the point (x, y)

$$(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

Gradient of PQ = $\dfrac{y_2 - y}{(x+h) - x}$

$$= \frac{(x+h)^4 - x^4}{h}$$

$$= \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$

$$= \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$

$$= 4x^3 + 6x^2h + 4xh^2 + h^3$$

$h = 0$ $\quad \dfrac{dy}{dx} = 4x^3$

Example

Obtain $\dfrac{dy}{dx}$ for each of the following

(i) $y = x^5$

(ii) $y = 3x^2$

(iii) $y = 5x - 3$

(i) $y = x^5$

$$\frac{dy}{dx} = 5x^4$$

(ii) $y = 3x^2$

$$\frac{dy}{dx} = 6x$$

(iii) $y = 5x - 3$

$$\frac{dy}{dx} = 5$$

The gradient of a tangent to a curve

The gradient of a tangent to a curve is given by the notation $\frac{dy}{dx}$

Given y as a function of x. $y = x^n$

Then The gradient of the tangent to the curve will be given by $\frac{dy}{dx} = nx^{n-1}$

The process of finding $\frac{dy}{dx}$ is called Differentiation (of y with respect to x)

The function thereby obtained is called the derived function or derivative (with respect to x)

In general

y	$\frac{dy}{dx}$
x^n	nx^{n-1}
ax^n	anx^{n-1}
ax	a
a	0

Example

Obtain $\frac{dy}{dx}$ given

(i) $y = x^3 + 4x^2 - 6x + 2$

(ii) $y = (x-3)^2$

(i) $y = x^3 + 4x^2 - 6x + 2$

$$\frac{dy}{dx} = 3x^2 + 8x - 6$$

(ii) $y = (x-3)^2$

$$y = x^2 - 6x + 9$$

$$\frac{dy}{dx} = 2x - 6$$

Example

(a) Differentiate $y = x^2 - 6x + 2$

(b) Differentiate $\frac{3}{x^2} + x^{\frac{5}{2}}$ with respect to x.

(a) $\frac{dy}{dx} = 2x - 6$

(b) $y = 3x^{-2} + x^{\frac{5}{2}}$

$$\frac{dy}{dx} = -6x^{-3} + \frac{5}{2}x^{\frac{3}{2}}$$

Questions

1. Obtain $\frac{dy}{dx}$ for each of the following

a) $y = x^7$ $\qquad \frac{dy}{dx} =$ ____________

b) $y = 3x^3$ $\qquad \frac{dy}{dx} =$ ____________

c) $y = 4x^2 - 5x + 2$ $\qquad \frac{dy}{dx} =$ ____________

d) $y = x^5 - x^2$ $\qquad \frac{dy}{dx} =$ ____________

e) $y = 7x + 3$ $\qquad \frac{dy}{dx} =$ ____________

2. (a) Obtain $\frac{dy}{dx}$ when $y = 3x^2 + 2x - 1$

(b) Hence obtain the value of $\frac{dy}{dx}$ when $x = 2$

a) $\frac{dy}{dx} =$ ____________

b) $\frac{dy}{dx} =$ ____________

3. (a) Obtain $\frac{dy}{dx}$ when $y = 6x^3 + x^2 - 11x - 6$

(b) Hence obtain the value of $\frac{dy}{dx}$ when $x = 2$

a) $\frac{dy}{dx} =$ ________________

b) $\frac{dy}{dx} =$ ________________

4. (a) Obtain $\frac{dy}{dx}$ when $y = x^3 - 3x^2 - 9x - 10$

(b) Hence obtain the values of x when $\frac{dy}{dx} = 0$

a) $\frac{dy}{dx} =$ ____________

b) $x =$ ____________