The Binomial expansion

Where n is a positive integer.

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

Gives as many terms of the expansion as may be required.

This may be adopted to expansions such as $(a+b)^n$ since its coefficients will be identical to those of $(1+x)^n$

Example

Expand $\left(1+x\right)^{10}$ as far as the fifth term

$$\left(1+x\right)^{10}=1+\left(10\right)\left(x\right)+\frac{\left(10\right)\left(9\right)}{2!}\left(x\right)^2+\frac{\left(10\right)\left(9\right)\left(8\right)}{3!}\left(x\right)^3+\frac{\left(10\right)\left(9\right)\left(8\right)\left(7\right)}{4!}\left(x\right)^4+\ldots$$

$$=1+10x+45x^2+120x^3+210x^4+\ldots$$

Example

Expand $(2-x)^5$

$$(2-x)^5 = 2^5\left(1-\frac{x}{2}\right)^5$$

$$= 32\left\{1+(5)\left(-\frac{x}{2}\right)+\frac{(5)(4)}{2!}\left(-\frac{x}{2}\right)^2+\frac{(5)(4)(3)}{3!}\left(-\frac{x}{2}\right)^3+\frac{(5)(4)(3)(2)}{4!}\left(-\frac{x}{2}\right)^4+\frac{(5)(4)(3)(2)(1)}{5!}\left(-\frac{x}{2}\right)^5\right\}$$

$$= 32\left\{1-\frac{5}{2}x+\frac{5}{2}x^2-\frac{5}{4}x^3+\frac{5}{16}x^4-\frac{1}{32}x^5\right\}$$

$$= 32\left\{32-80x+80x^2-40x^3+10x^4-x^5\right\}$$

Example

Expand as far as the term in x^3

$$(1-2x)^5(1+2x)^7$$

$$(1-2x)^5 = 1+(5)(-2x)+\frac{(5)(4)}{2!}(-2x)^2+\frac{(5)(4)(3)}{3!}(-2x)^3+\ldots$$

$$= 1-10x+40x^2-80x^3+\ldots$$

$$(1+2x)^7 = 1+(7)(2x)+\frac{(7)(6)}{2!}(2x)^2+\frac{(7)(6)(5)}{3!}(2x)^3+\ldots$$

$$= 1+14x+84x^2+280x^3+\ldots$$

$$(1-2x)^5(1+2x)^7 = (1-10x+40x^2-80x^3+\ldots)(1+14x+84x^2+280x^3+\ldots)$$

$$= 1+4x-16x^2-80x^3+\ldots$$

Example

a) Write down the expansion of $(1+x)^7$ in ascending powers of x up to and including the term in x^3.

b) Hence determine the value of 1.00001^7 correct to 15 decimal places.

$$(1+x)^7 = 1 + 7x + 21x^2 + 35x^3 + ...$$

$$x = 0.00001$$

$$1.00001^7 = 1 + 0.00007 + 0.0000000021 + 0.000000000000035$$
$$= 1.000070002100035$$