Conditional Probability

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Conditional probability is the probability of an event occurring, given that another event has already occurred.

P (A|B) means the probability of A occurring, given that B has occurred.

For two events A and B that are not independent,

$$P(A \cap B) = P(A) \times P(B|A)$$

Example:

A bag contains 2 blue balls and 2 red balls and two balls are selected randomly, if the first ball is removed without being replaced, calculate the probability that both balls will be red.

$$P(A \cap B) = P(A) \times P(B|A)$$

$$P(Red \cap Red) = P(Red) \times P(Red|Red)$$

$$= \frac{2}{4} \times \frac{1}{3}$$

$$= \frac{1}{6}$$

A calculation for conditional probability can be derived by simply re-arranging the formula:

$$P\ (A \cap B) = P\ (A) \times P\ (B|A)$$

$$P\ (B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P\ (A|B) = \frac{P(A \cap B)}{P(B)}$$

Note: The event that has already occurred is always on the bottom of the fraction

$$P\ (A|B) = \frac{P(A \cap B)}{P(B)}$$

Probability A and B happening at the same time

Probability B happening.

Probability of A occurring, given that B has occurred

Example:

A disc is selected from a bag containing 10 discs numbered individually 1 to 10.

Calculate the conditional probability that a prime number is selected given that the number selected is odd.

$$P\ (A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P\ (Prime|Odd) = \frac{P(Prime \cap Odd)}{P(Odd)}$$

$$= \frac{3/10}{5/10}$$

$$= \frac{3}{5}$$

This example could have been solved using a REDUCED sample method.

An Odd number has been selected. Now only five possible discs to select from.

Odd Prime numbers remaining are [3,5,7].

$$P\ (Prime|Odd) = \frac{3}{5}$$

Example:

The probability that a flight departs on time is 0.84, the probability that it arrives on time is 0.96

The probability that it both departs and arrives on time is 0.72

Find the probability that:

(i) Given that it departs on time, it will arrive on time.

(ii) Given that it arrives on time, it departed on time.

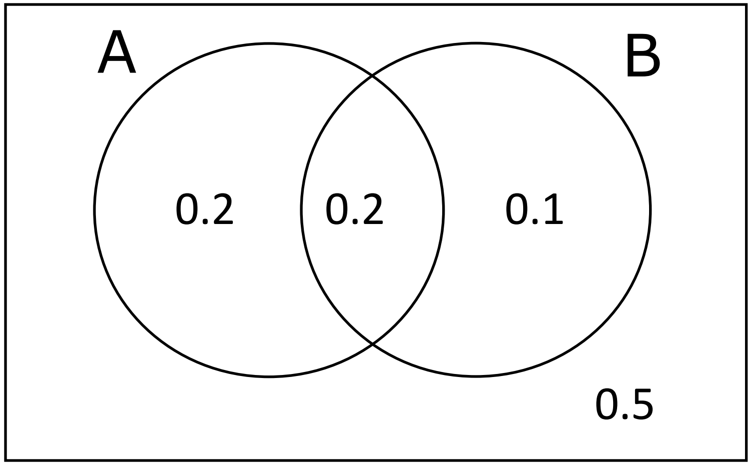
$$P\ (A|D) = \frac{P(A \cap D)}{P(D)} = \frac{0.72}{0.84} = \frac{6}{7}$$

$$P\ (D|A) = \frac{P(D \cap A)}{P(A)} = \frac{0.72}{0.96} = \frac{3}{4}$$

Note: P(A/D) and P(D/A) are not the same

Examples:

2. Two events A and B are such that $P(A) = 0.4$ $P(B) = 0.3$ $P(B|A) = 0.5$ Find:
   a) $P(A \cap B)$
   b) $P(A|B)$
   c) $P(A'|B')$



a)

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$0.5 = \frac{P(B \cap A)}{0.4}$$

$$P(B \cap A) = 0.5 \times 0.4$$
$$= 0.2$$

b)

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.2}{0.3} = \frac{2}{3}$$

c)

$$P(A' \cap B') = 0.5$$

$$P(B') = 0.7$$

$$P(A'|B') = \frac{P(A' \cap B')}{P(B')}$$

$$= \frac{0.5}{0.7} = \frac{5}{7}$$