The Poisson Approximation

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The Poisson distribution as an approximation of the Binomial.

When the number of trials in a Binomial distribution is very large, and the probability of success is very small,

$$\text{then } np \approx npq \text{ (as } q \approx 1)$$

therefore it is possible to change the distribution to a Poisson distribution.

We will only have an approximation of the probability.

$$X \sim Bin(n, p) \approx X \sim Po(\mathbf{np})$$

If n is large ($n > 50$), and p is small ($p < 0.1$)

Note: $\alpha = np$

In order to change from Binomial to the Poisson, we need to calculate the mean.

$$X \sim Bin\ (n = 500 , p = 0.01)$$

$$\text{Mean} = np = 500 \times 0.01 = 5$$

The approximation is therefore $X \sim Po\ (5)$

Example

X is the number of defective screws in a packet of 1000 screws.

X has a Binomial distribution such that;

$$X \sim Bin\ (\ n = 1000\ ,\ p = 0.003\ )$$

Calculate the probability that 2 or more of the screws are defective.

Since $X \sim Bin\ (\ 1000\ ,\ 0.003\ )$ and neither $n = 1000$ or $p = 0.003$ is in the Bin tables, we must use the Poisson approximation.

Mean = $np = 1000 \times 0.003 = 3$

$$X \sim Po(3)_{approx}$$

$$\begin{aligned} P(X \geq 2) &= 1 - P(\ X \leq 1) \\ &= 1 - 0.1991 \\ &= \underline{0.8009} \end{aligned}$$

Example 2

Given that X ~ Bin (150 , 0.04), find approximate values for

(a) $P(X \leq 3)$ (b) $P(X = 5)$ (c) $P(X > 4)$

Since X ~ Bin (150 , 0.04) $X \sim Po(6)_{approx}$

a) $P(X \leq 3) = \underline{0.1512}$

b) $P(X = 5) = P(X \leq 5) - P(X \leq 4)$

$= 0.4457 - 0.2851$

$= \underline{0.1606}$

c) $P(X > 4) = 1 - P(X \leq 4)$

$= 1 - 0.2851$

$= \underline{0.7149}$