Variance of a function of X

VAR[aX + b]

Example: (i) Find the mean and variance of the following D.R.V

X	0	1	2	3
P(X)	$^1/_3$	$^1/_2$	0	$^1/_6$

X	P(X)	X.P(X)	X^2.P(X)
0	$^1/_3$	0	0
1	$^1/_2$	$^1/_2$	$^1/_2$
2	0	0	0
3	$^1/_6$	$^3/_6$	$^9/_6$
		1	2

$$E[X] = 1$$

$$Var[X] = E[X^2] - (E[X])^2$$

$$= 2 - (1)^2$$

$$= 1$$

(ii) Find the mean and variance of the following functions of X:

a) 2x (b) 2x – 1 (c) 3x (d) 3x + 2

a)

2X	P(X)	X.P(X)	X^2.P(X)
0	$^1/_3$	0	0
2	$^1/_2$	$^2/_2$	$^4/_2$
4	0	0	0
6	$^1/_6$	$^6/_6$	$^{36}/_6$
		2	8

$$E[X] = 2$$

$$Var[X] = E[X^2] - (E[X])^2$$

$$= 8 - (2)^2$$

$$= 4$$

b)

2X – 1	P(X)	X.P(X)	X^2.P(X)
-1	$^1/_3$	$-^1/_3$	$^1/_3$
1	$^1/_2$	$^1/_2$	$^1/_2$
3	0	0	0
5	$^1/_6$	$^5/_6$	$^{25}/_6$
		1	5

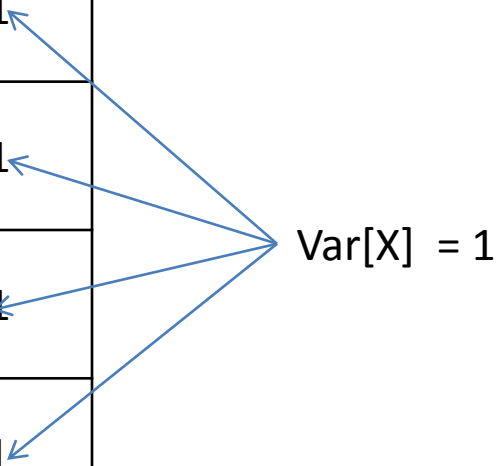
$$E[X] = 1$$

$$Var[X] = E[X^2] - (E[X])^2$$

$$= 5 - (1)^2$$

$$= 4$$

Function of x	Variance	N x Var[X]
2x	4	4 x 1
2x – 1	4	4 x 1
3x	9	9 x 1
3x + 2	9	x 1
aX + b	a^2	a^2 x Var[X]



For a linear function of X:

$$\text{Var}[aX + b] = a^2.\text{Var}[x]$$

This rule only works for linear functions, the variance of some simple quadratic functions or reciprocal functions may be calculated from an expanded table.

Example 2:

A random discrete variable X has a mean 10 and a standard deviation 3.

Find the mean and standard deviation of the following functions of X.

(a) $Y = X + 2$ (b) $Z = 3X$ (c) $W = 4X - 5$ (d) $V = 1 - 2X$

(e) $H = \frac{1}{4}(X - 3)$

Note:

We have been given the standard deviation in the question, therefore Variance is 3^2.

$$E[X] = 10 \text{ and } VAR[X] = 9$$

a) $Y = X + 2$

$E[Y] = E[X] + 2$
$= 10 + 2$
$= 12$

$VAR[Y] = 1^2.VAR[X]$
$= 1.\ 9$
$= 9$
$\underline{\sigma = 3}$

b) $Z = 3X$

$E[Z] = 3.E[X]$
$= 3 \times 10$
$= 30$

$VAR[Z] = 3^2.VAR[X]$
$= 9.\ 9$
$= 81$
$\underline{\sigma = 9}$

c) $W = 4X - 5$	$E[W] = 35$	$\sigma = 12$
d) $V = 1 - 2X$	$E[V] = -19$	$\sigma = 6$
e) $H = \frac{1}{4}(X - 3)$	$E[H] = 1.75$	$\sigma = {}^{3}/_{4}$

Summary of Rules for D.R.V.'s

Expectation: $E[X] = \Sigma\, X.P(X)$

$E[X^2] = \Sigma\, X^2.P(X)$

Variance: $Var[X] = E[X^2] - (E[X])^2$

For functions of X:

Linear Function: $E[aX + b] = a.E[X] + b$

$Var[aX + b] = a^2.Var[X]$

Quadratic Function:

$E[aX^2 + bX + c] = a.E[X^2] + b.E[X] + c$

Note 1: $E[X^2] \neq (E[X])^2$

Note 2: All other variations must be completed in an expanded table.