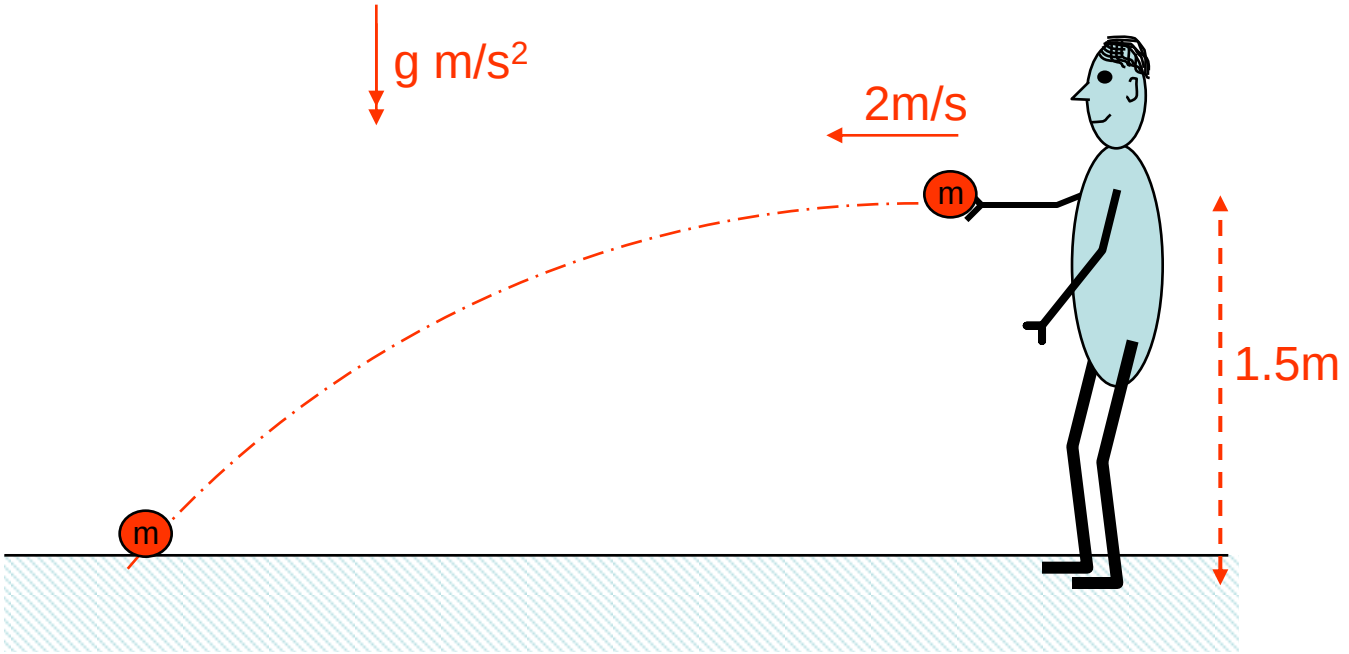
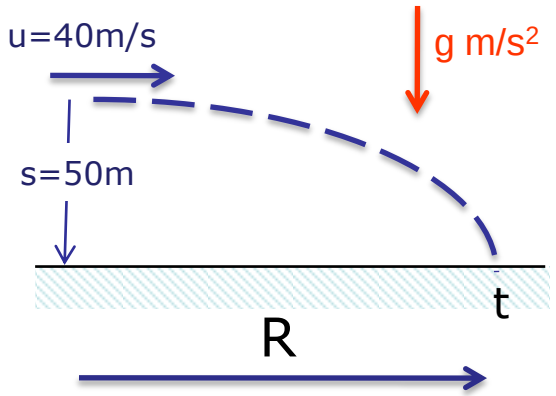
Projectiles



Here we are concerned with the motion of a particle projected in a medium in which the only active force is that due to gravity. i.e. its weight.

Example

A projectile is projected horizontally with speed 40 m/s from a point 50m above horizontal ground. Find the time at which the particle hits the ground and the horizontal range.



Vertical
u=0
s=−50
a=−g
t=?

$$s = ut + \frac{1}{2}at^2$$
$$-50 = -4.9t^2$$
$$t^2 = 10.20$$
$$t = 3.19\,\text{sec}$$

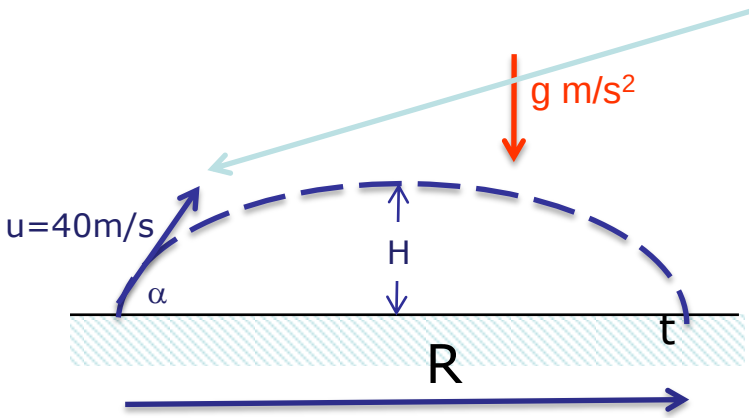
Horizontal
u=40
s=R
a=0
t=3.19

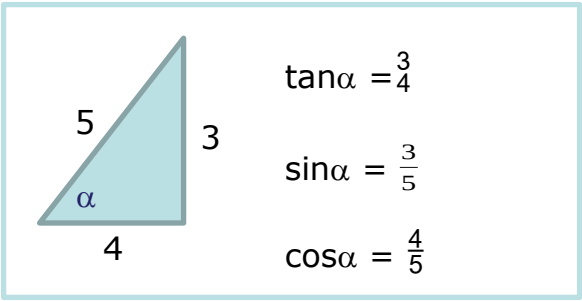
$$s = ut$$
$$R = 40 \times 3.19$$
$$R = 127.78m$$

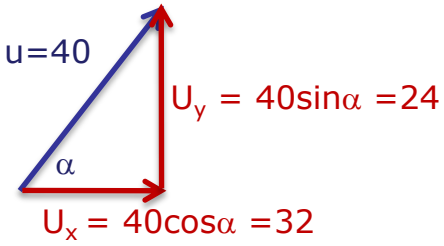
Example

A particle is projected with speed 40 m/s from a point A on horizontal ground at an angle of $\tan^{-1}\left(\frac{3}{4}\right)$ with the horizontal.

Find the range on the plane, the time of flight and the maximum height achieved and the time to achieve it. [Take g = 10 m/s^2]







Vertical
u=24
s=H
a=−g
v=0

t=?

$$v^2 = u^2 + 2as$$

$$0 = 24^2 - 20H$$

$$H = 28.8m$$

For maximum height v=0

$$s = ut + \frac{1}{2}at^2$$

$$0 = 24t - 5t^2$$

$$0 = t(24 - 5t)$$

$$t = 4.8\,\text{sec}$$

For range find t first when s=0
Then use horizontal motion

Horizontal
u=32
s=R
a=0
t=4.8

$$s = ut$$

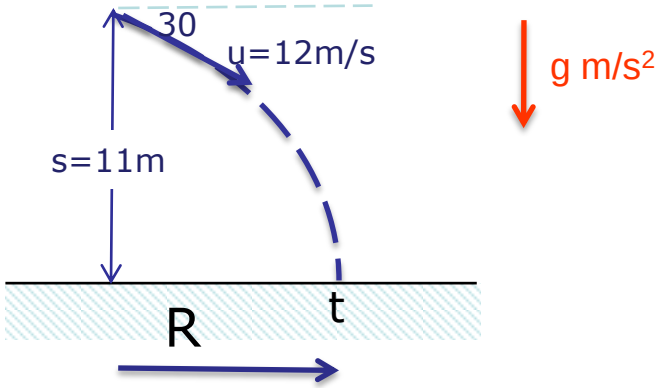
$$R = 32 \times 4.8$$

$$R = 153.6m$$

Example

A stone is thrown from the top of a tower which is 11m high and stands on horizontal ground. The speed of the projection is 12m/s and the initial direction of motion is at 30° below the horizontal.

Find the time taken for the stone to reach the ground and the range on the plane.



Vertical

u=12sin30 = 6
s=11
a=g
t=?

$$s = ut + \frac{1}{2}at^2$$

$$11 = 6t + 4.9t^2$$

$$4.9t^2 + 6t - 11 = 0$$

$$t = \frac{-6 \pm \sqrt{6^2 - 4 \times 4.9 \times -11}}{9.8}$$

$$t = 1.006 \text{sec}$$

Horizontal

u=12cos30 =10.39
s=R
a=0
t=4.8

$$s = ut$$

$$R = 10.39 \times 1.006$$

$$R = 10.458m$$

This provides a mathematical model for example for the motion of a cricket ball when thrown by a fielder or for a 'shot' projected by an athlete, etc… when frictional resistance is ignored.

The diagram shows the path (Trajectory) taken by a particle projected with a speed u at an angle θ to the horizontal, along a horizontal plane.

H = maximum height

R = range on the plane