Example

Express as a single fraction
$$\frac{3}{(x+2)} + \frac{2}{(x-1)}$$

Here the common denominator is (x+2)(x-1) $\frac{3}{(x+2)} + \frac{2}{(x-1)} = \frac{3(x-1)+2(x+2)}{(x+2)(x-1)}$ $= \frac{3x-3+2x+4}{(x+2)(x-1)}$ $= \frac{5x+1}{(x+2)(x-1)}$

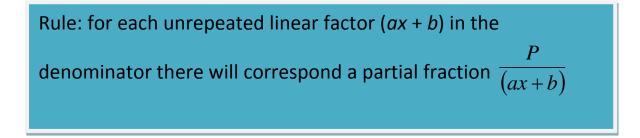
Denominator of 1^{st} fraction is multiplied by (x-1), hence multiply numerator by (x-1)Denominator of 2^{nd} fraction is multiplied by

(x+2), hence multiply numerator by (x+2)

Remove brackets on numerator and collect like terms.

Hence if we can take two fractions, or more, and write them as a single term. Then we must be able to perform the operation in reverse. This is known as placing a function as a sum of <u>PARTIAL FRACTIONS</u>

Partial Fractions



<u>Example</u>

Express in partial fractions $\frac{3x}{(2x-1)(x+1)}$

$$\frac{3x}{(2x-1)(x+1)} = \frac{A}{(2x-1)} + \frac{B}{(x+1)}$$

For some constant A and B.

By clearly the denominator we can use our identities, as the left hand side (LHS) must equate to the right hand side (RHS)

$$\frac{3x}{(2x-1)(x+1)} = \frac{A}{(2x-1)} + \frac{B}{(x+1)}$$
$$3x = A(x+1) + B(2x-1)$$

Look at the *x*. 3 = A + 2B

Look at the constant 0 = A - B $\therefore A = B$

Hence solving using simultaneous equations

$$3 = A + 2B$$

$$3 = 3A$$

$$A = 1 = B$$

$$\frac{3x}{(2x-1)(x+1)} = \frac{1}{(2x-1)} + \frac{1}{(x+1)}$$

<u>Example</u>

Express
$$f(x) = \frac{1+2x}{(2+x)(1-x)}$$
 in partial fractions

Hence (i) Evaluate
$$\int_{2}^{3} \frac{1+2x}{(2+x)(1-x)} dx$$

 (ii) Express f(x) as a power series in x as far as the term in x³ and give the range of convergence in the series.

$$\frac{1+2x}{(2+x)(1-x)} = \frac{A}{(2+x)} + \frac{B}{(1-x)}$$
$$1+2x = A(1-x) + B(2+x)$$

Look at the *x*. 2 = -A + B

Look at the constant 1 = A + 2B

Hence solving using simultaneous equations

$$3 = 3B$$
$$B = 1$$
$$A = 1 - 2 = -1$$

$$\frac{1+2x}{(2+x)(1-x)} = -\frac{1}{(2+x)} + \frac{1}{(1-x)}$$

Using partial fractions can make a function easier to integrate!

$$\int_{2}^{3} \frac{1+2x}{(2+x)(1-x)} dx = \int_{2}^{3} -\frac{1}{(2+x)} + \frac{1}{(1-x)} dx$$
$$= \left[-\ln(2+x) - \ln(1-x) \right]_{2}^{3}$$
$$= (-\ln(5) - \ln(-2)) - (-\ln(4) - \ln(-1))$$
$$= \ln 4 - \ln 5 - \ln 2$$
$$= \ln\left(\frac{2}{5}\right)$$

(ii)

(i)

$$-\frac{1}{(2+x)} = -\frac{1}{2} \left(1 + \frac{x}{2} \right)^{-1}$$

$$= -\frac{1}{2} \left\{ 1 + \left(-1 \right) \left(\frac{x}{2} \right) + \frac{(-1)(-2)}{2!} \left(\frac{x}{2} \right)^2 + \frac{(-1)(-2)(-3)}{3!} \left(\frac{x}{2} \right)^3 \right\}$$

$$= -\frac{1}{2} \left\{ 1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} \right\}$$

$$= -\frac{1}{2} + \frac{x}{4} - \frac{x^2}{8} + \frac{x^3}{16}$$

$$\frac{1}{(1-x)} = (1-x)^{-1}$$

$$= 1 + (-1)(-x) + \frac{(-1)(-2)}{2!} (-x)^2 + \frac{(-1)(-2)(-3)}{3!} (-x)^3$$

$$= \frac{1+x+x^2+x^3}{4}$$

$$\therefore \frac{1+2x}{(2+x)(1-x)} = -\frac{1}{(2+x)} + \frac{1}{(1-x)}$$
$$= \frac{1}{2} + \frac{5}{4}x + \frac{7}{8}x^2 + \frac{17}{16}x^3$$

The expression is valid for |x| < 1