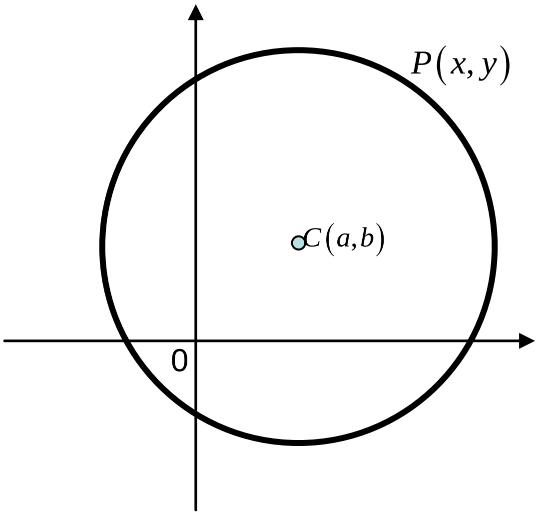
Equation of a circle through (a, b) with radius r



$$(x-a)^2 + (y-b)^2 = r^2$$

Example

Show that the equation $x^2 + y^2 - 2x + 4y - 4 = 0$ represents a circle and find its centre and radius.

This can be solved by completing the square twice!
Once for the terms in x and once for the terms in y.

$$x^2 + y^2 - 2x + 4y - 4 = 0$$
$$\left(x^2 - 2x + 1^2\right) + \left(y^2 + 4y + 2^2\right) = 4 + 1^2 + 2^2$$
$$(x-1) + (y+2)^2 = 9$$

Hence a circle with centre (1, -2) and radius 3.

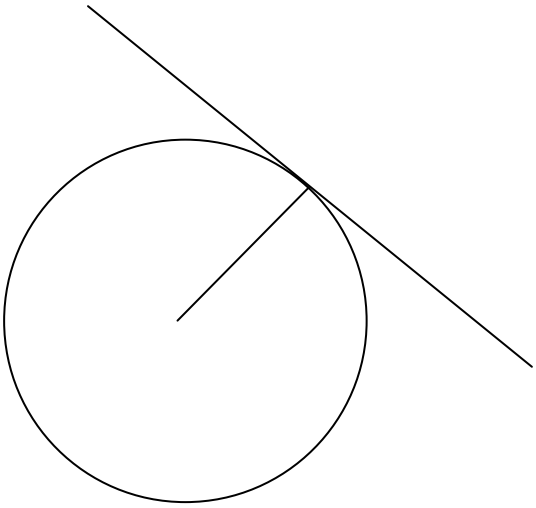
Example

Find the equation the circle with centre (–2, 4) and radius 4.

Using $(x-a)^2 + (y-b)^2 = r^2$

We can write the answer straight away!

$$(x+2)^2 + (y-4)^2 = 16$$

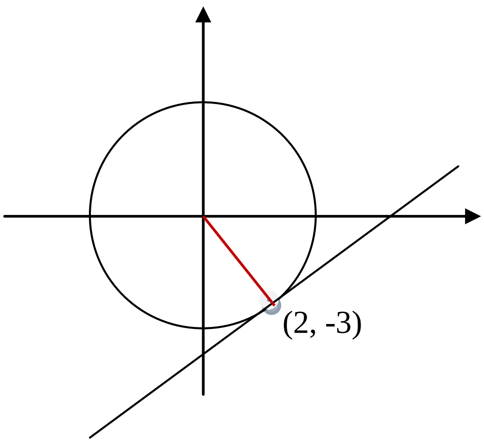


Angles between a tangent and its radius is always 90°

To find the equation of a tangent at a specific point on a circle, find the gradient of the radius, then the gradient of the tangent and use $y - y_1 = m(x - x_1)$

Example

Find the equation of the tangent to the circle $x^2 + y^2 = 13$ at the point $(2, -3)$



Gradient of the radius = $\frac{-3-0}{2-0} = -\frac{3}{2}$

Therefore gradient of tangent = $\frac{2}{3}$

Equation of the tangent

$$y - y_1 = m(x - x_1)$$

$$y + 3 = \frac{2}{3}(x - 2)$$

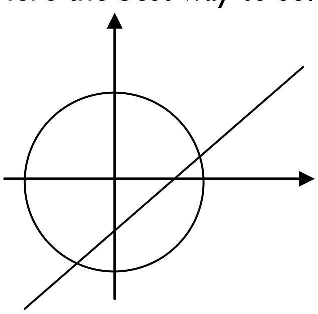
$$3y + 9 = 2x - 4$$

$$3y = 2x - 13$$

Example

Find the points of intersection of the circle $x^2 + y^2 = 20$ with the line $y = 3x - 2$

Here the best way to solve is using simultaneous equations



$$x^2 + y^2 = 20$$
$$x^2 + (3x-2)^2 = 20$$
$$x^2 + 9x^2 - 12x + 4 = 20$$
$$10x^2 - 12x - 16 = 0$$

We can simplify this a little further and then use factorisation!!

$10x^2 - 12x - 16 = 0$

$5x^2 - 6x - 8 = 0$

$(5x+4)(x-2) = 0$

So $x = 2$ or $x = -\frac{4}{5}$

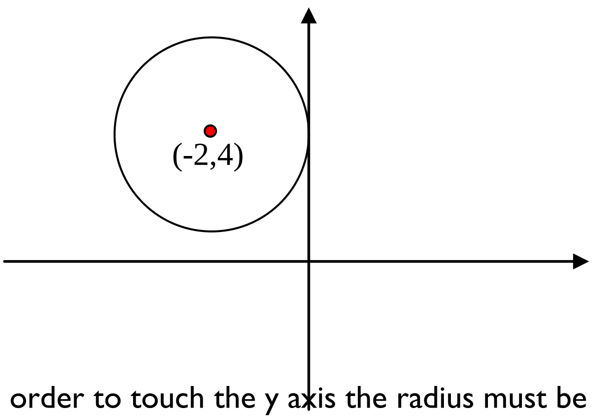
$x = 2$ means $y = 3 \times 2 - 2 = 4$ so crosses at $(2,4)$

$x = -\frac{4}{5}$ means $y = 3 \times -\frac{4}{5} - 2 = -\frac{22}{5}$ so crosses at $\left(-\frac{4}{5}, -\frac{22}{5}\right)$

<u>Example</u>

Find the equation of the circle centre (–2, 4) which touches the y axis.

Diagram will make this question much simpler



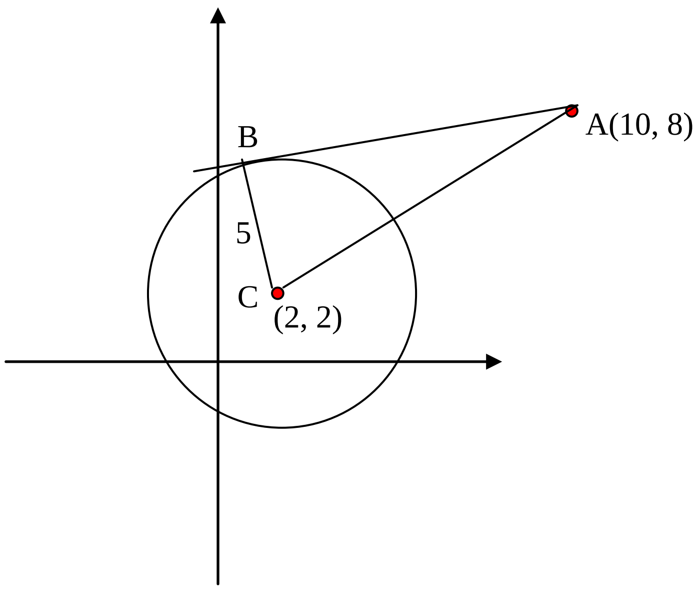
In order to touch the y axis the radius must be 2 units! Think about it..

Therefore $(x+2)^2 + (y-4)^2 = 4$

<u>Example</u>

AB is a tangent at B to the circle with centre C and equation $(x-2)^2 + (y-2)^2 = 25$. The point A has coordinate (10, 8). Find the area of the triangle ABC

Start by drawing a sketch of the circle given and the information we know.



The length AC is obtained from the sketch above using the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

AC = $\sqrt{(10-2)^2 + (8-2)^2} = \sqrt{8^2 + 6^2} = 10$

Using Pythagoras theorem.

AB = $\sqrt{10^2 - 5^2} = \sqrt{75} = 5\sqrt{3}$

Using formula for area of triangle half base times height.

Hence Area of triangle = $\frac{1}{2} \times 5 \times 5\sqrt{3} = \frac{25\sqrt{3}}{2}$

<u>1. A level past paper question</u>

The points (8, 4) and (2, 2) are the ends of the diameter of a circle C.

a) Find the equation of C.
b) Find the equation of the tangent to C at the point (8, 4)

<u>2. A level past paper question</u>

The circle with centre C has equation

$$x^2 + y^2 + 6x - 8y = 0$$

a) Find the radius of the circle and the coordinates of C.
b) The line y = x + 6 intersects the circle at the points P and Q. Calculate the length PQ.