Arithmetic Progression

The sequences of numbers

$a, a + d, a + 2d, a + 3d, \ldots$

are said to be an Arithmetic progression (AP)

Where

a = the first term

d = Difference between two consecutive terms

To Obtain the sum of n terms of an AP

The sum will be given by the addition of all terms

That is..

$$Sn = a + (a + d) + (a + 2d) + (a + 3d) + ... + (l - 2d) + (l - d) + l$$

Where l stands for the last term (or nth term) and is given by

$$l = a + (n - 1)d$$

Proof of formula for sum of n terms

$$Sn = a + (a+d) + (a+2d) + (a+3d) + \ldots\ldots + (l-2d) + (l-d) + l$$

$$Sn = l + (l-d) + (l-2d) + (l-3d) + \ldots\ldots\ldots + (a+2d) + (a+d) + a$$

$$2Sn = (a+l) + (a+l) + (a+l) + \ldots\ldots\ldots + (a+l) + (a+l)(a+l)$$

$$2Sn = n(a+l)$$

since $l = a + (n-1)d$

$$2Sn = n(a+l)$$

$$= n(a + a + (n-1)d)$$

$$Sn = \frac{n}{2}(2a + (n-1)d)$$

The sequences of numbers

a, $a + d$, $a + 2d$, $a + 3d$, …

With first term a and common difference d has

n^{th} term (or last term) given by

$$U_n = a + (n-1)d$$

And sum of n terms given by

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Example

Interpret $\sum_{r=1}^{n} r$ and find an expression for the sum.

$$\sum_{r=1}^{n} r = 1+2+3+4+.....+n$$

Here we have an arithmetic progression with a = 1, d = 1 and n

$$\sum_{r=1}^{n} r = Sn = \frac{n}{2}\left[2\times 1+(n-1)\times 1\right]$$

$$= \frac{n}{2}(2+n-1)$$

$$= \frac{n}{2}(n+1)$$

Example

How many terms of the sequence 2, 5, 8, …. Should be taken for the sum to exceed 1000?

From the above we have a = 2, d = 3 and $S_n > 1000$

$$S_n = \frac{n}{2}\left[2\times 2+(n-1)\times 3\right]$$

$$= \frac{n}{2}(4+3n-3)$$

$$= \frac{n}{2}(3n+1) > 1000$$

$$n(3n+1) > 2000$$

$$3n^2 + n > 2000$$

$$3n^2 + n - 2000 > 0$$

$$n = \frac{-1\pm\sqrt{1-4\times 3\times -2000}}{6}$$

$$= \frac{-1+154.922}{6}$$

$$= 25.6$$

$$\therefore n = 26$$

Example

Given that the 10th term of an AP is 45 and that the sum of the first 10 terms is 270 find

a) The first term

b) The common difference

c) The sum of the first 15 terms

a) 10th term $U_{10} = 45$ $a + 9d = 45$

Sum of 10 terms $S_{10} = 270$ $\frac{10}{2}[2a + 9d] = 270$

$a + 9d = 45$

$2a + 9d = 54$

$a = 9$

b) $9 + 9d = 45$

$d = 4$

c) $S_{15} = \frac{15}{2}[2 \times 9 + 14 \times 4]$

$= 555$

Example

An arithmetic progression has first term 3 and common difference *d*. The nth term is 93 and the sum of the first *n* terms is 768. Find the values of *n* and *d*.

a) 10th term $U_n = 93$ $3+(n-1)d=93$

Sum of 10 terms $S_n = 768$ $\frac{n}{2}[6+(n-1)d]=768$

$$(n-1)d=90$$

$$\frac{n}{2}[6+90]=768$$

$$48n=768$$

$$n=16$$

$$15d=90$$

$$d=6$$

Example

The first term of an arithmetic series is 3. The seventh term is twice the third term.

a) Find the common difference.

b) Calculate the sum of the first 20 terms of the series.

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Example

The tenth term of an arithmetic series is 34 and the sum of the first forty terms is 2620. Find the first term and the common difference of the series.

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