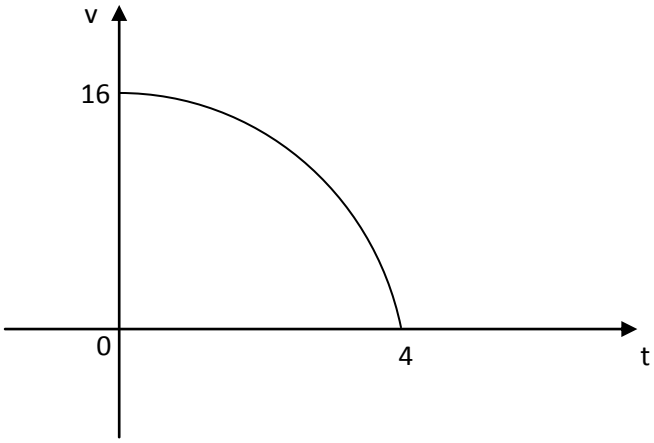
Kinematics: Variable Acceleration

Example

Given that the velocity of a particle changes with time such that $v = 16 - t^2$

a) Draw a velocity-time graph for the first 4 seconds of motion
b) Obtain the distance covered in the first 4 seconds
c) Workout the acceleration when (i) t = 1 seconds
(ii) t = 3 seconds

a)



b) Distance travelled in the first 4 seconds = area under the graph (Integration finds the area under a curve)

Distance covered = $\int_0^4 16 - t^2 dt$

$= \left[16t - \frac{t^3}{3}\right]_0^4$

$= \left(16 \times 4 - \frac{4^3}{3}\right) - (0)$

$= 42\frac{2}{3}$ m

c) (i) $a = -2t$

$t = 1 \quad a = -2\text{m/s}^2$

(ii) $t = 3 \quad a = -6\text{m/s}^2$

Essentially this shows use that with acceleration written as a function of time we have a varied acceleration with time.

Furthermore we can see in the above example that given v,

$a = \frac{dv}{dt}$ and $s = \int v dt$

Example

Given that a particle moves from a point 0 with velocity $v = 3t^2 - 6t + 3$ and that when $t = 1$, $s = 2$ m, find expressions for

a) the acceleration of the particle
b) the displacement of the particle, in terms of *t*.

a) $a = \dfrac{dv}{dt}$

$a = 6t - 6$

b) $s = \int v dt$

$s = t^3 - 3t^2 + 3t + c$

When t =1, s=2

$2 = 1 - 3 + 3 + c$

$c = 1$

$s = t^3 - 3t^2 + 3t + 1$

Example

A particle has displacement from point 0 given by $s = t^3 - t^2 + 4t$

a) Obtain the displacement from 0 at time $t = 2$

b) Obtain an expression the velocity and acceleration of the particle at time

(i) *t* (ii) t = 2.

a) $s = 2^3 - 2^2 + 4(2) = 8 - 4 + 8 = 12m$

b) (i) $v = 3t^2 - 2t + 4$ $\quad a = 6t - 2$

$v = 3(2)^2 - 2(2) + 4$ $\quad a = 6(2) - 2$

(ii) $= 12 - 4 + 4$ $\quad a = 12 - 2$

$= 12$ $\quad = 10$

In general

Given displacement, *s*, as a function of *t*

$v = \frac{ds}{dt}$ and $a = \frac{dv}{dt}$

Example

Abody moves with varied acceleration $a = 6t - 4$ and at time $t = 0$ the body is moving with velocity 3m/s and has displacement 5m from the origin.

Find in terms of t expressions for the velocity and displacement of the particle.

$$v = \int (6t - 4)dt$$

$$= 3t^2 - 4t + c$$

t=0, v=3

$$3 = c$$

$$v = 3t^2 - 4t + 3$$

$$s = \int (3t^2 - 4t + 3)dt$$

$$= t^3 - 2t^2 + 3t + c$$

t=0, s=5

$$5 = c$$

$$s = t^3 - 2t^2 + 3t + 5$$

In general

Given the acceleration, a, as a function of t

$v = \int a\,dt$ and $s = \int v\,dt$

1. A level past paper question

A particle P is projected from the origin 0 so that it moves along the x-axis. At time t s after projection, the velocity of the particle, v ms^{-1}, is given by

$$v = 3t^2 - 24t + 45$$

a) Show that P first comes to instantaneous rest when t = 3.
b) Find an expression for the acceleration of P at time t s.
c) Find an expression for the displacement of P in the first 3 seconds of its motion.
d) Find the distance travelled by the particle in the first 3 seconds of its motion.

a) $v = 0$

$$0 = 3t^2 - 24t + 45$$
$$0 = 3(t^2 - 8t + 15)$$
$$0 = 3(t-3)(t-5)$$

$\therefore t = 5, t = 3$

Hence P comes to rest at $t = 3$ seconds for the first time.

b) $a = 6t - 24$

c) $x = \int 3t^2 - 24t + 45dt$

$x = t^3 - 12t^2 + 45t + c$

$t = 0, x = 0 \quad \therefore c = 0$

$x = t^3 - 12t^2 + 45t$

d) $t = 3 \quad x = (3)^3 - 12(3)^2 + 45(3) = 54$ metres