Interval Estimates

Suppose that we have a population with an unknown mean μ , and with variance σ^2 known.

An estimate for μ is obtained through a large sample (size n) and $\bar{x}$ is calculated.

Where $\bar{x} \in \bar{X}$ and $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

Next, the standardised values of z corresponding to this value of $\bar{x}$ will be given by

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

and since 95% of all observations of z lie between ± 1.96 we can write $-1.96 < z < 1.96$

which with some rearranging gives

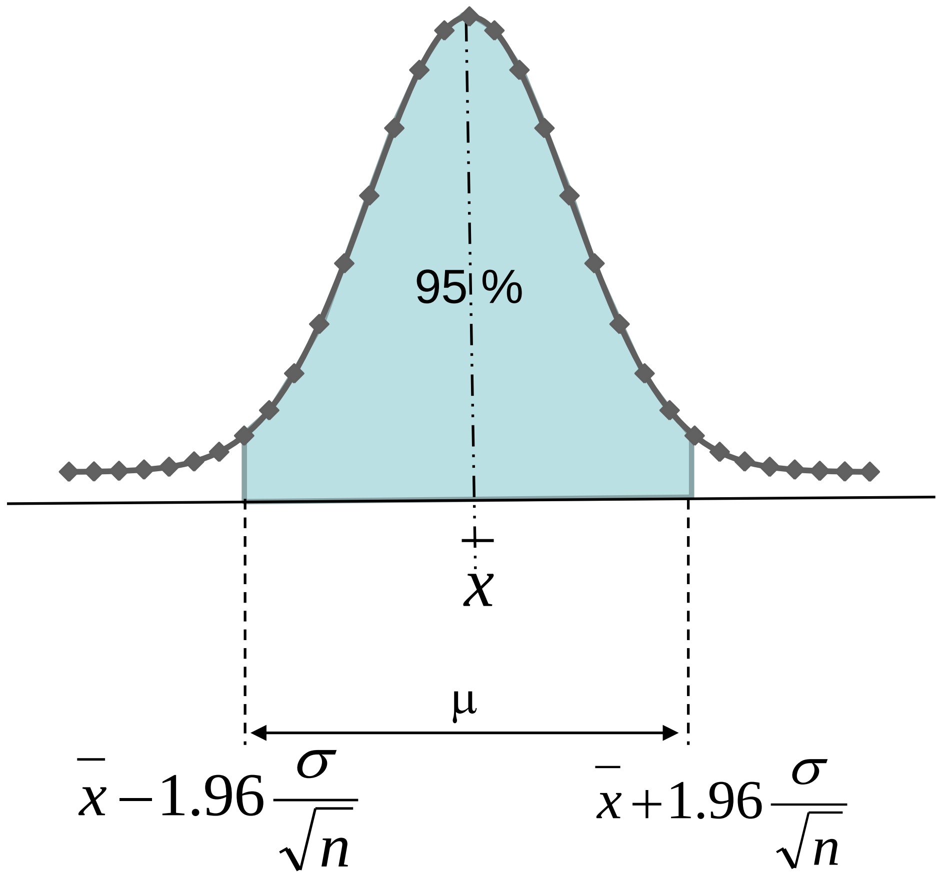
$$\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}$$

This interval $\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$ is known as the 95%

symmetric Confidence Interval for the true value of μ.

It must be remembered that μ is not a variable, but is fixed, albeit unknown.

Consider the distribution $\bar{x}$ of with this fixed but unknown value of the mean μ, and our point estimate of $\bar{X}$ at the centre of the interval estimate



Hence by 95% confidence we understand that the interval is 95% certain to contain μ.

Other confidence intervals are readily obtained

90% Confidence $\bar{x} \pm 1.645 \frac{\sigma}{\sqrt{n}}$ (smaller interval)

99% Confidence $\bar{x} \pm 2.575 \frac{\sigma}{\sqrt{n}}$ (larger interval)

In other words the greater degree of confidence required then the larger the interval. However large sample sizes reduces the width of interval.

To find an N% symmetric Confidence Interval for μ:

$$\mu = \bar{x} \pm z.\frac{\sigma}{\sqrt{n}}$$

This interval is N% certain to contain μ based on the sample mean.

Example

In a strictly controlled test on a certain type of tennis ball, the height of bounce X is approximately normally distributed with variance 4 cm^2.

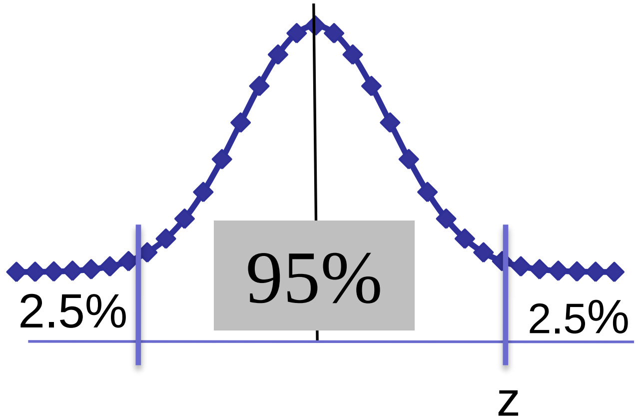
Obtain 95% and 99% confidence intervals for the mean height of bounce when

a) a sample of 30 tennis balls is tested to give a mean of 140.7cm

b) a sample of 60 tennis balls is tested to give a mean of 141.1 cm.

95% Interval = [140.6, 141.6] 99% Interval = [140.4, 141.8]

a) 95% Symmetric CI



P(Z < z) = 0.975

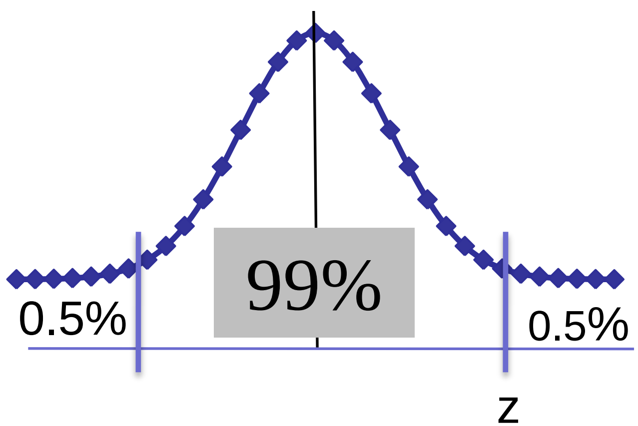
z = 1.96

$$\mu = \bar{x} \pm z.\frac{\sigma}{\sqrt{n}}$$

$$\mu = 140.7 \pm 1.96.\frac{2}{\sqrt{30}}$$

95% Interval: μ = [140.0, 141.4]

b) 99% Symmetric CI



P(Z < z) = 0.995

z = 2.576

$$\mu = \bar{x} \pm z.\frac{\sigma}{\sqrt{n}}$$

$$\mu = 140.7 \pm 2.576.\frac{2}{\sqrt{30}}$$

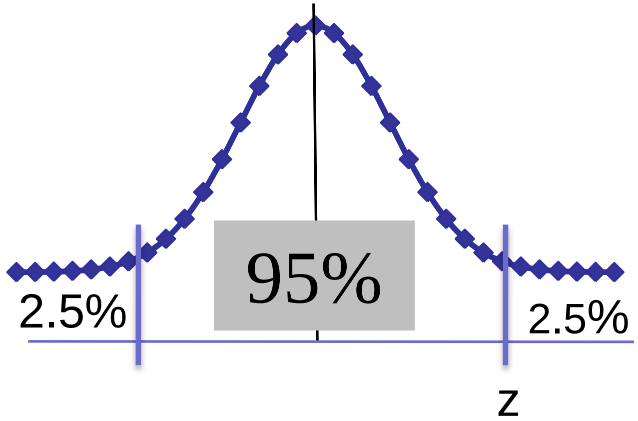
95% Interval: μ = [139.8, 141.6]

Example

A soft drinks machine is tested for the amount dispensed by taking a sample of 20 drinks. The results give a sample mean of 236 ml and a known variance of 144 ml^2. Obtain a 95% confidence interval for the amount of drink dispensed on average by this machine.

X = Amount of drink dispensed
$X \sim N(236, 144)$ [Note: $\sigma = 12$]

95% Symmetric CI



$P(Z < z) = 0.975$
$z = 1.96$

$$\mu = \bar{x} \pm z.\frac{\sigma}{\sqrt{n}}$$

$$\mu = 236 \pm 1.96.\frac{12}{\sqrt{20}}$$

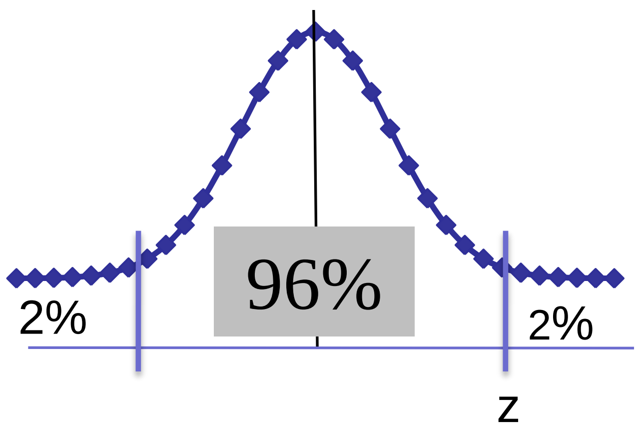
95% Interval: $\mu = [230.7, 241.3]$

<u>Example</u> (if required)

A random sample of 100 men is taken and the mean height is found to be 180cm. The population variance is $49cm^2$. Find the 96% confidence interval for μ, the population mean height of men.

X = Mean Height
X ~ N (180, 49) [Note: $\sigma = 7$]

96% Symmetric CI



P(Z < z) = 0.98
z = 2.054

$$\mu = \bar{x} \pm z.\frac{\sigma}{\sqrt{n}}$$

$$\mu = 180 \pm 2.054.\frac{7}{\sqrt{100}}$$

95% Interval: μ = [181.4, 178.6]